HW 2

Alec Johnson

$\boxed{\S 13.5}$

(1) $\vec{r}(t) = \begin{pmatrix} 3\sin t \\ 3\cos t \\ 4t \end{pmatrix}$

$\vec{r}'(t) = \begin{pmatrix} 3\cos t \\ -3\sin t \\ 4 \end{pmatrix}$

$\vec{r}''(t) = \begin{pmatrix} -3\sin t \\ -3\cos t \\ 0 \end{pmatrix}$

$\vec{r}'''(t) = \begin{pmatrix} -3\cos t \\ 3\sin t \\ 0 \end{pmatrix}$

(recall from Section 13.4)

$|\vec{r}'(t)| = \sqrt{3^2\cos^2 t + 3^2\sin^2 t + 4^2}$

$= \sqrt{3^2 + 4^2}$

$= 5$

$\hat{T} = \dfrac{\vec{r}'(t)}{|\vec{r}'(t)|} = \dfrac{1}{5}\begin{pmatrix} 3\cos t \\ -3\sin t \\ 4 \end{pmatrix}$

$\kappa \hat{N} = \dfrac{d\hat{T}}{ds} = \dfrac{1}{|r'|}\dfrac{d\hat{T}}{dt}$

$= \dfrac{3}{25}\begin{pmatrix} -\sin t \\ -\cos t \\ 0 \end{pmatrix}$

$\kappa = |\kappa \hat{N}| = \dfrac{3}{25}$

$\hat{N} = \begin{pmatrix} -\sin t \\ -\cos t \\ 0 \end{pmatrix}$

$\hat{B} = \hat{T} \times \hat{N}$

$= \dfrac{1}{5}\begin{pmatrix} 3\cos t \\ -3\sin t \\ 4 \end{pmatrix} \times \begin{pmatrix} -\sin t \\ -\cos t \\ 0 \end{pmatrix}$

(1)

$\boxed{\hat{B} = \dfrac{1}{5}\begin{pmatrix} 4\cos t \\ -4\sin t \\ -3 \end{pmatrix}}$ (2)

$\vec{r}' \times \vec{r}'' = \begin{pmatrix} 3\cos t \\ -3\sin t \\ 4 \end{pmatrix} \times 3\begin{pmatrix} -\sin t \\ -\cos t \\ 0 \end{pmatrix}$

$= 3\begin{pmatrix} 4\cos t \\ -4\sin t \\ -3 \end{pmatrix}$ ← another way to get $\hat{B}$ is to rescale this vector to have unit length.

$|r' \times r''| = 15$

$\tau = \dfrac{(r' \times r'') \cdot r'''}{|r' \times r''|^2}$

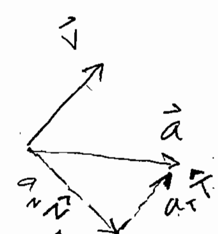
$= \dfrac{3\begin{pmatrix} 4\cos t \\ -4\sin t \\ -3 \end{pmatrix} \cdot 3\begin{pmatrix} -\cos t \\ \sin t \\ 0 \end{pmatrix}}{15^2}$

$= \dfrac{9 \cdot (-4)}{15^2}$

$= \dfrac{-4}{5^2}$

$\boxed{\tau = \dfrac{-4}{25}}$ (2)

(11) $\vec{r}(t) = \begin{pmatrix} t+1 \\ 2t \\ t^2 \end{pmatrix}$



Write $\vec{a} = a_T \hat{T} + a_N \hat{N}$ (*)
at the value $t = 1$.

Solution

$\vec{v} = \vec{r}'(t) = \begin{pmatrix} 1 \\ 2 \\ 2t \end{pmatrix}$, $\vec{a} = \vec{r}''(t) = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

We take the dot product of (*) with $\vec{v}$:

$\vec{a} \cdot \vec{v} = (a_T \hat{T} + a_N \hat{N}) \cdot \vec{v}$

$= a_T |\vec{v}|$.

So $a_T = \dfrac{\vec{a} \cdot \vec{v}}{|v|}$

$= \dfrac{\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2t \end{pmatrix}}{\sqrt{1 + 4 + 4t^2}}$

$= \dfrac{4t}{\sqrt{5 + 4t^2}}$

When $t = 1$, $\boxed{a_T = \dfrac{4}{3}}$ (3)

$a_N^2 = |a|^2 - a_T^2$ (by pythagorean theorem)

$= 2^2 - \left(\dfrac{4}{3}\right)^2$

$= \dfrac{20}{9}$

$\boxed{a_N = \dfrac{2}{3}\sqrt{5}}$ (2)

$\vec{a} = \dfrac{4}{3}\hat{T} + \dfrac{2\sqrt{5}}{3}\hat{N}$