1. **Problem 4, Assadi's fall 2007 midterm 1.** Find the partial derivatives $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$, where

$$w(x, y) = \cos(x^2 - y) \text{ and}$$
$$x(s, t) = 2s\cos(t), \quad y(s, t) = t^2 \sin(2s)$$

   (a) by using the Chain rule (express your answer in terms of s and t) and
   (b) by plugging the equations for x and y into w and computing the partial derivatives directly.

2. **Problem 5, Assadi's fall 2007 midterm 1.** Find the equation of the tangent plane and the normal line for the surface S given by $x^2+3y^2-25z^2 = 13$ at the point $\mathbf{p} = (-1, 2, 0)$.

3. **Problem 5, Wilson's fall 2004 midterm 2. Local extrema.** Find and *identify* all relative maxima, relative minima, and saddle points for the function $f(x, y) = x^3 + y^2 - 6x^2 + y - 1$. For each critical point write the second-order Taylor expansion near the critical point.

4. **Problem 7, Assadi's fall 2007 midterm 1.** Let $f(x, y) = 4 - x^2 - y^2$.

   (a) Sketch the level sets $f(x, y) = k$ for $k = -5, 0, 3$ and 4.
   (b) Sketch the graph of $z = f(x, y)$.
   (c) Find a unit vector $\mathbf{v}$, in the direction in which $f(x, y)$ *decreases* most rapidly at the point $\mathbf{p} = (1, 2\sqrt{2})$. What is the rate of change in this direction?
   (d) Sketch the vector $\mathbf{v}$ at the point $\mathbf{p}$ on the graph in part (a). What do you observe?

5. **Problem 2, Wilson's fall 2004 midterm 1.** Let $\mathbf{r}(t) = (3\sin t, 3\cos t, 4t)$. describe the motion of an object along a curve in space. Find as functions of t:

   (a) the velocity,
   (b) the acceleration,
   (c) the unit tangent vector,
   (d) the principal unit normal vector,
   (e) the curvature,
   (f) the tangential (scalar) component of the acceleration, and
   (g) the normal (scalar) component of the acceleration.

6. **Problem 1, Assadi's spring 2007 midterm 1.** Suppose that an object P is moving so that its position vector at time t is given by $X(t) = (t + e^t, t - e^{-t}, t^2)$.

   (a) Find the velocity $V(t)$ and the acceleration vectors $A(t)$ of P at $t = 1$.
   (b) Now consider the curve described by the velocity $V(t)$. Find the curvature of *this curve* $V(t)$ at $t = 1$.

7. **Problem 1, Assadi's spring 2005 midterm 2.** Find all points $P = (x, y, z)$ at which the function $f(x, y, z) = 2x + 3y + z + 5$ attains a minimum subject to the constraint $g(x, y, z) = 4x^2+9y^2-z = 0$.

8. **Problem 1, Assadi's fall 2007 final.** The plane $x + y + 4z = 2$ cuts the cone $z^2 = x^2 + y^2$ in an ellipse. Using the method of Lagrange multipliers, find the greatest and the smallest values that the function $f(x, y, z) = z^2$ takes on the ellipse. Where do these values occur?

9. Find the dimensions of the cylinder of greatest volume which fits inside a sphere of radius R. (Hint: this is a constrained optimization problem.)

10. Find the maximum and minimum values of $f(x, y) = x^3 + y^3$ subject to the constraint that $x^2 + y^2 \leq 4$.

11. **Problem 2, Wilson fall 2004 midterm 2.** A certain can is intended to have a radius of $r = 2$ inches and a height of $h = 5$ inches. Due to manufacturing tolerances, the radius changes to $r = 2.01$ inches and the height changes to $h = 4.98$ inches. Use partial derivatives to approximate how much the volume of the can changes from its disigned value. (Do not just compute two volumes and subtract.)