Test 1, #3

$y'' + 10y' + 25y = \frac{e^{-5t}}{t} + 25 \ , \ t > 1 \qquad y(1) = 0, \ y'(1) = 0$

$y'' + 10y' + 25y = 0$

$r^2 + 10r + 25 = 0$

$(r+5)^2 = 0$

$r = -5$ **(5 pts)**

$y_1(t) = e^{-5t}, \ y_2(t) = te^{-5t}$

treat $\frac{e^{-5t}}{t}$, 25 separately

$y_p(t) = 1$ (particular sol'n for 25) **(5 pts)**

$y_p(t) = u_1(t)e^{-5t} + u_2(t)te^{-5t}$

$$W(e^{-5t}, te^{-5t}) = \begin{vmatrix} e^{-5t} & te^{-5t} \\ -5e^{-5t} & e^{-5t} - 5te^{-5t} \end{vmatrix}$$

$= e^{-10t} - 5te^{-10t} + 5te^{-10t}$

$= e^{-10t}$

$$\left.\begin{aligned} u_1'(t) &= \frac{-\frac{e^{-5t}}{t} \cdot te^{-5t}}{e^{-10t}} = -1 \\ u_1(t) &= -t \end{aligned}\right\}$$

but te^{-5t} is part of the homogeneous sol'n, so this term is not needed.

$$\left.\begin{aligned} u_2'(t) &= \frac{e^{-5t}/t \cdot e^{5t}}{e^{-10t}} = \frac{1}{t} \\ u_2(t) &= \ln t \end{aligned}\right\} \textbf{(5 pts)}$$

so $\left. y(t) = c_1e^{-5t} + c_2te^{-5t} + (\ln t)\, te^{-5t} + 1 \right\}$ **(5 pts)**

$y'(t) = -5c_1e^{-5t} + c_2e^{-5t} - 5c_2e^{-5t} + e^{-5t} + (\ln t)e^{-5t} - 5(\ln t)\, te^{-5t}$

$y(1) = 0 \Rightarrow 0 = c_1e^{-5} + c_2e^{-5} + 1$

$y'(1) = 0 \Rightarrow 0 = -5c_1e^{-5} + c_2e^{-5} - 5c_2e^{-5} + e^{-5}$

$c_1 + c_2 = -e^5$

$-5c_1 - 4c_2 = -1$

$$\left.\begin{aligned} c_1 &= 4e^5 + 1 \\ c_2 &= -5e^5 - 1 \end{aligned}\right\} \textbf{(5 pts)}$$

$$\boxed{y(t) = (4e^5 + 1)e^{-5t} + (-5e^5 - 1)te^{-5t} + (\ln t)te^{-5t} + 1}$$

(5 pts)