3. For the system

$$\frac{dx}{dt} = (4-y)(x+y)$$

$$\frac{dy}{dt} = (2+y)(x-y)$$

(a) Determine all critical points

ANS.	(10 pts)

(b) Find corresponding linear system near each critical points.

ANS.	(10 pts)

$\underline{a}$ $\dot{x}=0$ $4=y$ or $x=-y$

$\dot{y}=0$ $y=-2$ or $x=y$

i) $\rightarrow$ $4=y=-2$ not poss

(ii) $\rightarrow$ $x=y=4$

(iii) $\rightarrow$ $(2,-2)$

(iv) $\rightarrow$ $(0,0)$

5

5

b) $DF = \begin{bmatrix} 4-y & -x-y+4-y \\ 2+y & x-y-2-y \end{bmatrix}$

$= \begin{bmatrix} 4-y & 4-x-2y \\ 2+y & x-2y-2 \end{bmatrix}$ 6

(i) $DF(0,0) = \begin{bmatrix} 4 & 4 \\ 2 & -2 \end{bmatrix}$ (ii) $DF(2,-2) = \begin{bmatrix} 6 & 6 \\ 0 & 4 \end{bmatrix}$

(iii) $DF(4,4) = \begin{bmatrix} 0 & -8 \\ 6 & -6 \end{bmatrix}$ 3

1 $(X-X_0)^{\cdot} = DF(X_0)(X-X_0)$

$\boxed{\text{or}}$ $u = x - x_0$ $(0,0)$ $\begin{pmatrix} x \\ y \end{pmatrix}^{\cdot} = \begin{pmatrix} 4 & 4 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ 2

$x = u+2$
$y = v-2$ 1
$(2,-2)$

$\dot{u} = (4-v+2)(u+2+v-2) = (6-v)(u+v) = \begin{pmatrix} 6 & 6 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} + O_2(u,v)$

$\dot{v} = (v)(u+2-v+2) = 4v + uv - v^2$

4

$x = u+4$
$y = v+4$

$\dot{u} = -v(u+4+v+4) = -8v - uv - v^2 = -8v$

$\dot{v} = (v+6)(u-v) = 6u - 6v + uv - v^2 = 6u - 6v + O_2(u,v)$

4

(c) Find the eigenvalues of each linear system. What conclusions can you draw about the nonlinear system near each critical point?

(asymptotically stable or unstable node, unstable saddle, asymptotically stable or unstable spiral, center)

ANS.

(10 pts)

(i) $\begin{vmatrix} \lambda-4 & -4 \\ -2 & \lambda+2 \end{vmatrix} = \lambda^2 - 2\lambda - 8 - 8$
$= (\lambda - 1)^2 - 17$
$\lambda = 1 \pm \sqrt{17}$
$\lambda_1 < 0 \quad \lambda_2 > 0$

2

d)

$\rightarrow \begin{bmatrix} -3 \pm \sqrt{17} & -4 \\ -2 & 3 \pm \sqrt{17} \end{bmatrix} \phi = 0$

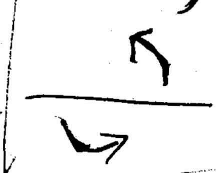
$\phi = \begin{bmatrix} 3 \pm \sqrt{17} \\ 2 \end{bmatrix}$

(ii) $\lambda_1 = 6 \quad \lambda_2 = 4 \quad > 0$ 1 $\rightarrow \phi_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \phi_2 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

(iii) $\begin{vmatrix} \lambda & 8 \\ -6 & \lambda+6 \end{vmatrix} = \lambda^2 + 6\lambda + 48$
$= (\lambda + 3)^2 + 39$
$\lambda = -3 \pm i\sqrt{39} \quad Re(\lambda) < 0$

$\rightarrow \dot{x} = -8y$

spiral runs counterclockwise



2

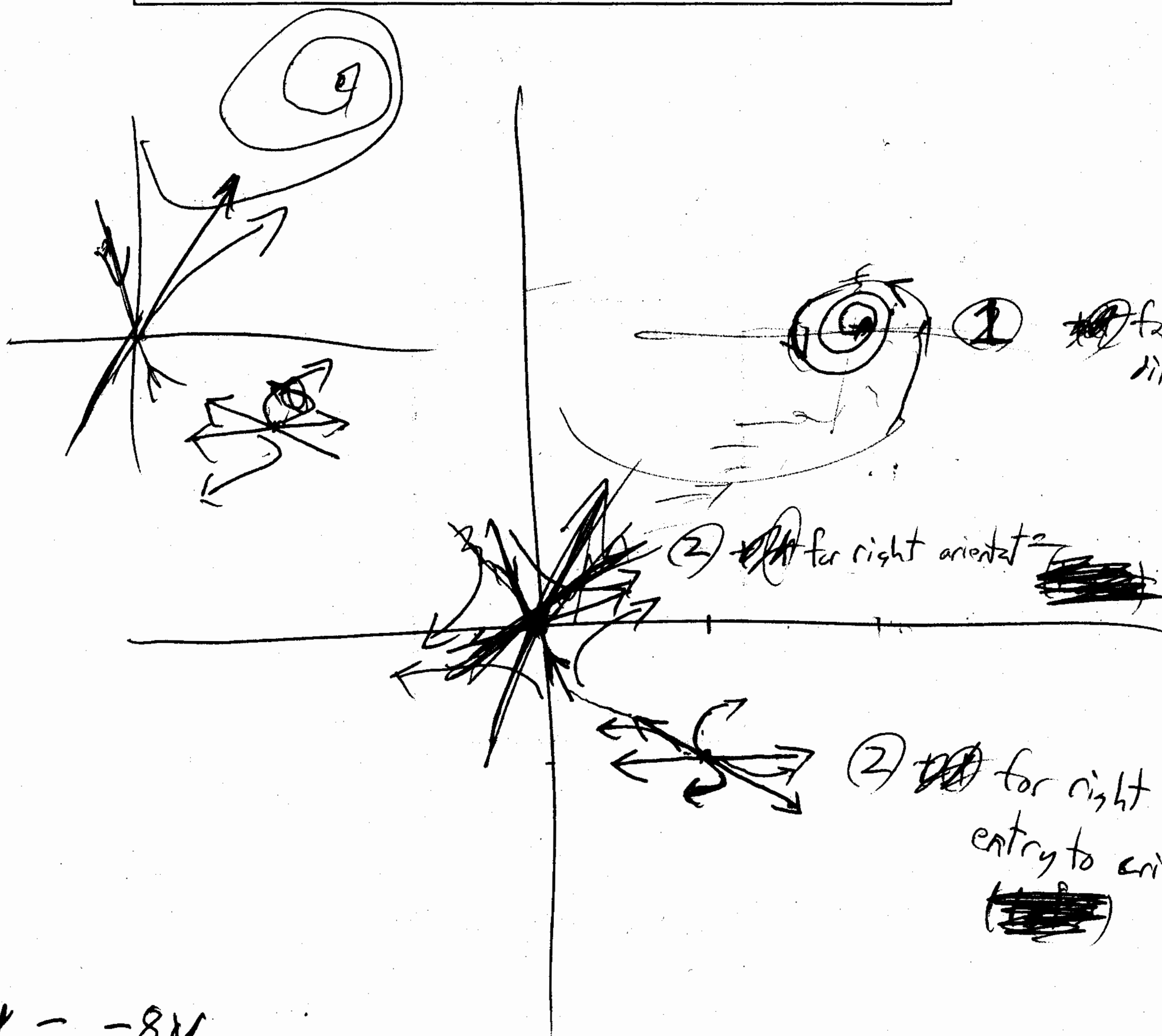
5 i) unstable saddle
ii) unstable node
iii) stable spiral

part d 5

(d) Sketch a phase portrait for the nonlinear system.

ANS.

(10 pts)



1 ~~pt~~ for right direct^n [redacted]

2 ~~pt~~ for right orientat^n [redacted]

2 ~~pt~~ for right entry to critical pt. [redacted]

$\dot{x} = -8y$